Advanced Higher Homework Exercises **ANSWERS**

Binomial Theorem

1. Expand $\left(p+2q\right)^6$

$${}^6C_0p^6+{}^6C_1p^5\left(2q\right)+{}^6C_2p^4\left(2q\right)^2+{}^6C_3p^3\left(2q\right)^3+{}^6C_4p^2\left(2q\right)^4+{}^6C_5p\left(2q\right)^5+{}^6C_6\left(2q\right)^6$$

$$=p^6+12p^5q+60p^4q^2+160p^3q^3+240p^2q^4+192pq^5+64q^6$$

- Binomial coefficients
- Ascending powers of *p* and descending powers of *q*
- Simplified answer

2. Expand $\left(2u-3c\right)^5$

$${}^5C_0(2u)^5-{}^5C_1(2u)^4(3c)+{}^5C_2(2u)^3(3c)^2-{}^5C_3(2u)^2(3c)^3+{}^5C_4(2u)(3c)^4-{}^5C_5(3c)^5$$

$$=32u^5-240u^4c+720u^3c^2-1080u^2c^3+810uc^4-243c^5$$

- Binomial coefficients and alternating signs
- Powers of (2*u*) and (3*c*)
- Simplified answer

3. Find the constant term in the expansion of $\left(x+\frac{2}{x}\right)^8$

$${}^8C_4x^4\left(\frac{2}{x}\right)^4=\frac{1120x^4}{x^4}=1120$$

- Correct combination
- Powers of *x*
- Answer

4. Write down and simplify the general term in the expansion of $\left(x^2+\frac{1}{x}\right)^{10}$.

Hence or otherwise, obtain the term in x^{14}.

$${}^{10}C_r(x^2)^{10-r}\left(\frac{1}{x}\right)^r={}^{10}C_rx^{20-3r}$$

$20-3r=14$

$r=2$

$${}^{10}C_2x^{20-3x2}=45x^{14}$$

- General term
- Simplify fully power of *x*
- Obtain value for *r*
- Answer

5. Use the binomial theorem to evaluate $(0.96)^6$ correct to 4 significant figures.

$(1-0.04)^6 = 1-{}^6C_1(0.04)+{}^6C_2(0.04)^2-{}^6C_3(0.04)^3+{}^6C_4(0.04)^4-\ldots$

$= 1-6\times 0.04+15\times 0.0016-20\times 0.000064+15\times 0.00000256-\ldots$

$= 1-0.24+0.0240-0.00128+0.00003840-\ldots$

$= 1.0240384-0.24128$

$= 0.7827584\ldots$

$= 0.7828(4sf)$

- Appropriate binomial expression
- Expansion
- Convincing numerical detail
- Correctly rounded answer based on previous steps.

6. Prove that for all k ≥ 3, $\binom{k+2}{3}-\binom{k}{3}=k^2$.

$$\binom{k+2}{3}-\binom{k}{3}=\frac{(k+2)!}{(k+2-3)!3!}-\frac{k!}{(k-3)!3!}=\frac{(k+2)!}{(k-1)!3!}-\frac{k!}{(k-3)!3!}$$

$$=\frac{(k+2)(k+1)k}{3!}-\frac{k(k-1)(k-2)}{3!}$$

$$=\frac{k(k^2+3k+2)-k(k^2-3k+2)}{6}$$

$$\frac{6k^2}{6}$$

$$k^2$$

- Definitions of $\binom{k+2}{3} and \binom{k}{3}$
- Simplify factorials
- Begin to expand brackets
- Complete proof